

The *Species Motrix*

After his “success” with the Platonic solids, Kepler decided at some point to try to develop a system of astronomy based on *physical* principles. Perhaps he was inspired by Tycho’s demolition of the celestial spheres. Without solid spheres to move the planets around, what makes them move? Perhaps the same kind of thing that moves things here on Earth. Kepler attempted to treat the motion of the planets as though it conformed with Aristotelian *sublunary* physics. This means he adopted Aristotle’s idea that $v = kF/m$, where v is the speed of an object, F is the force applied to the object, and m is related to the size and density of the object (it’s a bit like what we call mass, or quantity of matter, but it plays a different role in the theory).¹ We will refer to the quantity represented by m using the Latin term Kepler used: *moles*. Finally, k is just a proportionality constant.

1. To get a handle on the force F , Kepler needed to examine the speeds of celestial objects. Use the table below to calculate the (average) speed of each planet (in AU/year), as determined in the Copernican system.

Planet		Orbital Period (in years)	Orbital Radius (in AU)	Speed (in AU/yr)
Mercury	☿	0.24	0.47	
Venus	♀	0.62	0.74	
Earth	♁	1	1	
Mars	♂	1.88	1.5	
Jupiter	♃	11.7	5.2	
Saturn	♄	29.1	7.9	

2. What do you notice about the speeds of the planets?
 - (a) They are all the same.
 - (b) The farther a planet is from the Sun, the faster it moves.
 - (c) The farther a planet is from the Sun, the slower it moves.
 - (d) There is no pattern at all to the speeds.
3. Does this necessarily imply (based on Aristotelian physics) that the force on a planet is smaller if the planet is farther from the Sun? Why or why not?

¹The modern term “mass” refers to an object’s resistance to *changes* in its motion, whereas for Kepler m measured the resistance to motion itself.

4. Recall that Copernicus ridded astronomy of the equant, but in order to do so he had to introduce small epicycles into the motion of the planets. Kepler, the most devout Copernican of his day, brought the equant back. In the first part of his *Astronomia Nova* Kepler reformulated the Copernican theory using equants insted of the extra epicycle that Copernicus used. So for Kepler, the planets moved in circles but in a non-uniform way. What do you think was Kepler's motivation for bringing back the equant?

5. With this construction of the orbits (eccentric circles with equants) in place, Kepler examined how the speed of a planet would change with its distance from the center. Go to a computer and run the `KeplerEquant` simulation. This simulation shows the eccentric/equant construction that Kepler used to describe the motions of the planets. Two plots show the distance (r) of the planet from the central point, as well as the planet's speed (v), as a function of time. How are the changes in distance related to the changes in speed?

6. Record the below the maximum distance of the planet from the center, as well as the speed it has at that distance. Also record the minimum distance and the corresponding speed.

max dist. = _____ speed = _____

min dist. = _____ speed = _____

7. Show that the product of distance and speed gives you the same value in each case. Note that if this holds true for *all* points on the orbit, then it implies $rv = k \rightarrow v = k/r$ (ie speed is inversely proportional to distance).

8. If the speed of a *single planet* varies inversly with its distance from the center, can we then conclude (using Aristotlian physics) that the force on a planet is inversely proportional to the distance of the planet from the Sun? Why or why not?

9. Kepler became convinced that the force on a planet varied inversly with its distance from the center point. Recall that in Copernicus' system there is nothing at the center point (which is the center of Earth's orbit). For Kepler, who was seeking a physical explanation for the planets' motion, it was unthinkable that the location that determines the force on a planet could be unoccupied by any body. Since he was a Copernican, what body did he decide to place at this central point?

10. Kepler decided that the force that moves the planets, which he called the *species motrix*, must emanate from the Sun. Probably inspired by William Gilbert's treatise on the magnet, *De Magnete*, he thought that the Sun exerted something like a magnetic force on the planets. The primary action of this force would be to push the planets around in their orbits (remember, in Aristotlian physics there is no motion without a force). To explain this Kepler assumed the Sun must rotate, which would cause its tendrils of force to sweep the planets along in their orbits. To see what this looks like run the `SpeciesMotrix` simulation. The simulation shows a planet orbiting the Sun in an eccentric orbit. The orange lines represent the *species motrix*. Plots of the planet's distance from the Sun and its speed as a function of time are shown. Is the planet held fixed to one *species motrix* line, or does it move through the lines?
11. How is the density of the *species motrix* lines related to the speed of the planet? In other words, is the force on the planet greater when the *species motrix* lines are close together or when they are far apart? Explain your answer.
12. What happens if we compare different planets? Are the speeds of the planets inversely proportional to their distances? Copy the speeds you calculated above into the table below and then compute the product of average distance from the Sun and average speed for each planet. Record your results in the table below.

Planet		Orbital Radius (AU)	Speed (AU/yr)	Radius \times Speed (AU ² /yr)
Mercury	☿	0.47		
Venus	♀	0.74		
Earth	♁	1		
Mars	♂	1.5		
Jupiter	♃	5.2		
Saturn	♄	7.9		

13. If the force really does vary inversely with distance, as Kepler thought, then what do the above results imply about the *moles* of the planets? Rank the planets in order of increasing *moles*. To better understand this you may want to play around with the `KeplerInertia` program. This shows two planets orbiting the Sun. Their orbital radii and *moles* are adjustable. The *species motrix* supplies a force that is inversely proportional to the distance of the planet from the Sun. Try adjusting the *moles* and leaving the radius (and thus the force) the same to see what happens.